A Computer Aided Design Technique for Hybrid and Monolithic Microwave Amplifiers Employing Distributed Equalizers with Lumped Discontinuities

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Abstract — **This paper will addres the use of mixed lumped and distributed elements in the matching equalizers of microwave amplifers for hybrid and monolithic MIC realizations. In this work we show how the computer aided real frequency technique can be extended to design broadband amplifiers employing distributed equalizers with lumped discontinuities. The scattering based two-variable description of lossless equalizers with mixed lumped-distributed elements will be discussed and the potential benefits of the approach will be indicated by examples.**

I. INTRODUCTION

Because of the need for miniaturization of large-scale microwave systems, there is a great deal of effort being directed towards the area of Microwave Integrated Circuits (MIC) in hybrid or monolithic form. MIC design of amplifiers require that active and passive device models are fully developed and the circuit-design approach is through and well disciplined. In the design process, besides the active devices, refined models are needed for treatment of device to circuit medium interfaces incorporating parasitic effects and junction discontinuities. In this regard, utilization of mixed lumped and distributed circuits to model the front-end, back-end and interstage equalizers of an amplifier and the interconnects, would offer advantages for accurate simulation of MIC layouts, where the physical sizes, parasitics and discontinuities are naturally embedded in the design process.

In the well known real frequency design techniques, the complex variable $p = \sigma + j\omega$ is employed in the descriptive network functions, which yields lumped element circuit components in matching equalizers. Utilizing the hybrid or monolithic integrated circuit production technologies, it is possible to built lumped circuit elements up to 10 GHz. Beyond these frequencies however, physical sizes must be included in the design process. In this case, equal delay transmission lines are employed in the designs, where the complex variable p is replaced with the Richard variable $\lambda = \Sigma + j\Omega$, where $\Omega = \tan\omega\tau$ and τ specifies the equal delay length of transmission lines. For a more accurate simulation of the MIC layout, mixed lumped and distributed elements need to be used, where the physical connection of lumped elements can be covered with transmission lines and parasitic of the discontinuities can be imbedded into lumped elements. In this case, it is necessary to carry out all the designs in at least two variables namely, p for lumped elements and λ for equal delay transmission lines.

In this work we show how the computer aided real frequency technique for broadband amplifiers can be extended to design distributed equalizers with lumped discontinuities. Considering a general amplifier configuration with front-end and back-end equalizers of mixed lumped-distributed element type, the proposed design procedure involves the following major steps:

The active device is assumed to be characterized by a set of measured scattering data. The mixed element equalizers on the other hand are described by the scattering functions in two complex frequency variables, namely the conventional p for the lumped elements and λ for the transmission lines. Then, the formulation of the simplified real frequency algorithm is extended to the two-variable description of the equalizers, where the partially defined polynomials describing the scattering functions in two-varibles are constructed by optimization of the transducer power gain of the overall amplifier structure together with matching equalizers.

II. CONSTRUCTION OF TWO-VARIABLE SCATTERING FUNCTIONS FOR LOSSLESS TWO-PORTS FORMED WITH LUMPED AND DISTRIBUTED ELEMENTS

A typical distributed equalizer network with lumped discontinuities can be modeled in the generic form of a lossless two-port formed with cascade connections of lumped elements and commensurate transmission lines (Unit Elements), where the lumped elements represent the discontinuities in the cascade (Fig.1).

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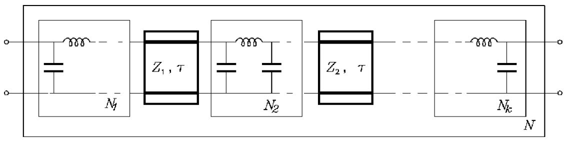

Fig.1. Generic form of mixed lumped-distributed two-ports

The scattering matrix describing the mixed lumped-distributed element two-port can be expressed in the Belevitch canonical form as [1],

$$S(p,\lambda)=\frac{1}{g(p,\lambda)}\begin{pmatrix} h(p,\lambda) & \sigma f(-p,-\lambda) \\ f(p,\lambda) & -\sigma h(-p,-\lambda) \end{pmatrix}, \qquad (1)$$

where

- $f(p,\lambda)$, $g(p,\lambda)$ and $h(p,\lambda)$ are real polynomials in the complex variables p and λ, ($\lambda = \tanh p\tau$, τ being the delay length of unit elements).
- $g(p,\lambda)$ is a Scattering Hurwitz polynomial,
- $f(p,\lambda)$ is monic and σ is unimodular constant
- $$g(p,\lambda)g(-p,-\lambda) = h(p,\lambda)h(-p,-\lambda)+f(p,\lambda)f(-p,-\lambda) \qquad (2)$$

Let the real polynomials $g(p,\lambda)$ and $h(p,\lambda)$ with partial degrees n_p and n_λ be expressed as

$$g(p,\lambda)=\overline{p}^T\Lambda_g\overline{\lambda} \text{ and } h(p,\lambda)=\overline{p}^T\Lambda_h\overline{\lambda}, \qquad (3)$$

where $\overline{p}^T = \left[1\, p\, p^2 \cdots p^{n_p}\right]$, $\overline{\lambda}^T = \left[1\lambda\lambda^2\cdots\lambda^{n_\lambda}\right]$ and

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & g_{1n_\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p0} & g_{n_p1} & \cdots & g_{n_pn_\lambda} \end{bmatrix}, \Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & h_{1n_\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p0} & h_{n_p1} & \cdots & h_{n_pn_\lambda} \end{bmatrix}$$

For the cascade topology under consideration, the scattering matrix and hence the canonical polynomials $f(p,\lambda)$, $g(p,\lambda)$ and $h(p,\lambda)$ have to satisfy some additional independent conditions to ensure the realizability as a passive lossless cascade structure. In this regard, the followings may immediately be remarked:

a) For the mixed element lossless two-port, the polynomial $f(p,\lambda)$ defines the transmission zeros of the cascade and it is given by

$$f(p,\lambda)=f_0(p)f_1(\lambda) \text{ ,} \qquad (4)$$

where, $f_0(p)$ and $f_1(\lambda)$ contains the transmission zeros due to the lumped sections and unit elements in the cascade, respectively. If n_λ unit elements are considered in cascade mode, then $f_1(\lambda)=(1-\lambda^2)^{n_\lambda/2}$. In most of the practical cases, it is appropriate to choose $f_0(p)$ as an even/odd real polynomial, which corresponds to reciprocal lumped structures. A particularly useful case is obtained for $f_0(p)=1$, which corresponds to a typical low-pass structure having transmission zeros only at infinity. In this case the polynomial $f(p,\lambda)$ takes the simple form

$$f(p,\lambda)=(1-\lambda^2)^{n_\lambda/2} \text{ .} \qquad (5)$$

Another practical case is to choose $f_0(p)=p^{n_p}$, which corresponds to a typical high-pass structure.

b) When the transmission lines are removed from the cascade structure, one would end up with a lumped network whose scattering matrix can fully be described independently in terms of the canonical real polynomials $f_0(p)$, $g_0(p)$ and $h_0(p)$ as in the Belevitch representation (1) and (2). This would correspond to the boundary case where we set $\lambda = 0$ in the scattering description given by (1) and (2). In other words, the boundary polynomials $h(p,0), g(p,0)$ and $f(p,0)$ define the cascade of lumped sections which take place in the composite structure, where $g(p,0)$ is strictly Hurwitz and

$$g(p,0)g(-p,0)=h(p,0)h(-p,0)+f(p,0)f(-p,0) \qquad (6)$$

c) When the lumped elements are removed from the cascade structure, one would obtain cascade connection of transmission lines. In this case the resulting distributed prototype whose transmission zeros are defined as $f_1(\lambda)=(1-\lambda^2)^{n_\lambda/2}$ can fully be described independently in terms of three canonical real polynomials $f_1(\lambda)$, $g_1(\lambda)$ and $h_1(\lambda)$ as in the Belevitch representation (1). For the particular case of low-pass type lumped sections described by (5), setting $p=0$ in the scattering description given by (1) results in the boundary polynomials $f_1(\lambda)=f(0,\lambda)$, $g_1(\lambda)=g(0,\lambda)$ and $h_1(\lambda)=h(0,\lambda)$. In this case, the boundary polynomials $h(0,\lambda), g(0,\lambda)$ and $f(0,\lambda)$ define the cascade of UEs which take place in the composite structure, where $g(0,\lambda)$ is strictly Hurwitz and

$$g(0,\lambda)g(0,-\lambda)=h(0,\lambda)h(0,-\lambda)+(1-\lambda^2)^{n_\lambda} \qquad (7)$$

If the lumped sections in the cascade are assumed to be of high-pass type, setting $p=\infty$, we end up with the boundary polynomials $h(\infty,\lambda)$ and $g(\infty,\lambda)$. In this case the paraunitary relation (7) is modified as,

$$g(\infty,\lambda)g(\infty,-\lambda)=h(\infty,\lambda)h(\infty,-\lambda)+(1-\lambda^2)^{n_\lambda}, \qquad (8)$$

where $g(\infty,\lambda)$ is strictly Hurwitz . The more general cases with finite transmission zeros in lumped sections can also be treated following a similar reasoning.

d) The single variable boundary polynomials $h(p,0), g(p,0)$ related by (6) and $h(0,\lambda)$, $g(0,\lambda)$

satisfying the relation (7) define the first row and the first column entries of Λ_h and Λ_g matrices. Now the problem is to generate the remaining unknown entries, which carry the cascade connection information so that the two-variable paraunitary relation (2d) is satisfied together with the boundary conditions introduced in (a) and (b).

A. Fundamental Equation Set

For the generation of canonic polynomials $f(p,\lambda)$, $g(p,\lambda)$ and $h(p,\lambda)$ in realizable form, the solution of the two-variable paraunitary relation is essential. By equating the coefficients of the same powers of the complex frequency variables in the equality (2), we end up with the following set of equations:

$$g_{0,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l} g_{0,l}g_{0,2k-l} = h_{0,k}^2 + f_{0,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}(h_{0,l}h_{0,2k-l} + f_{0,l}f_{0,2k-l}) \quad \vdots \; (k=0,1,.....,n_\lambda) \tag{9a}$$

$$\sum_{j=0}^{i}\sum_{l=0}^{k}(-1)^{i-j-l} g_{j,l}g_{i-j,2k-1-l} = \sum_{j=0}^{i}\sum_{l=0}^{k}(-1)^{i-j-l}\left[h_{j,l}h_{i-j,2k-1-l} + f_{j,l}f_{i-j,2k-1-l}\right] \quad \vdots \; (i=1,3,...,2n_p-1 \;\;,\; k=0,1,.....,n_\lambda-1) \tag{9b}$$

$$\sum_{j=0}^{i}(-1)^{i-j}\left(g_{j,k}g_{i-j,k} + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{j,l}g_{i-j,2k-l}\right) = \sum_{j=0}^{i}(-1)^{i-j}\begin{pmatrix} h_{j,k}h_{i-j,k} + f_{j,k}f_{i-j,k} + \\ 2\sum_{l=0}^{k-1}(-1)^{k-l}\left[h_{j,l}h_{i-j,2k-l} + f_{j,l}f_{i-j,2k-l}\right]\end{pmatrix} \quad \vdots \; (i=2,4,...,2n_p-2 \;\;,\; k=0,1,.....,n_\lambda) \tag{9c}$$

$$g_{n_p,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{n_p,l}g_{n_p,2k-l} = h_{n_p,k}^2 + f_{n_p,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}(h_{n_p,l}h_{n_p,2k-l} + f_{n_p,l}f_{n_p,2k-l}) \quad \vdots \; (k=0,1,.....,n_\lambda) \tag{9d}$$

The solution of the above equation set for the coefficients g_{ij} of $g(p,\lambda)$ is equivalent to the factorization of a two-variable polynomial, and therefore, the equation set in (9) will be referred to as the Fundamental Equation Set (FES). In order to end up with a realizable system, the necessary constraints leading to an acceptable particular solution must be established. This requires additional coefficient constraints reflecting the connectivity information for each class of cascade topology in order to end up with a unique solution of FES. In this context, by straightforward analysis, the coefficient constraints leading to an explicit solution of FES can easily be generated for symmetric structures or those regular structures in which the lumped sections are confined to be simple low-pass, high-pass or band-pass elements [2]-[3].

Based on the above discussion, we end up with the following semi analytic procedure to construct two-variable canonical polynomials:

Procedure:

- Assuming a regular cascaded structure as in Fig. 1, select the number of lumped and distributed elements (n_p, n_λ) and the polynomial $f(p,\lambda)$.
- Choose the coefficients of the polynomials $h(p,0)$ and $h(0,\lambda)$ (or $h(\infty,\lambda)$) as the independent parameters and generate the Hurwitz polynomials $g(p,0)$ and $g(0,\lambda)$ (or $g(\infty,\lambda)$.) using (6) and (7) (or 8) respectively.
- In addition to the boundary conditions stated above, establish further topologic constraints on the coefficients for each class of regular cascade topology by analysis.
- Utilizing the boundary conditions and the coefficient constraints in FES obtain the unknown coefficients.

B. Construction of regular ladders with Unit Elements

From the physical implementation point of view, practical circuit configurations of common interest consist of commensurate lines incorporating lumped discontinuities in low-pass or high-pass type lumped elements. For those type regular structures, the proposed semi-analytic approach has been successfully applied and two-variable characterization for a variety of practical mixed element circuits have been obtained [3]-[4].

III. EXTENSION OF REAL FREQUENCY TECHNIQUE TO DESIGN AMPLIFIERS WITH MIXED ELEMENT EQUALIZERS

In the design of broadband microwave amplifiers, the fundamental problem is to realize lossless interstage equalizers as well as front-end and back-end matching networks so that the transfer of power from source to load is maximized over a prescribed frequency band (Fig.2).

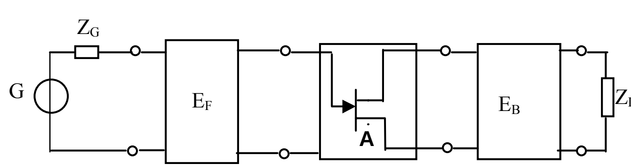

Fig.2. Single stage amplifier with front-end and back-end equalizers

The well known Simplified Real Frequency Technique (SRFT), which is based on the scattering description of the equalizers and the active device provides an efficient tool for the design of microwave amplifiers [5]-[6].

The extension of the real frequency technique for designing equalizers with mixed elements require a unique characterization of the matching network in terms of a number of independent free parameters. The proposed procedure in the previous section resides on the generation of two-variable scattering functions for the lumped-distributed cascaded networks. This procedure readily leads us to the two-variable generalization of the SRFT, which can be outlined as follows:

- In the description of mixed element two-ports, the real normalized scattering parameters are generated from the partially defined numerator polynomial $h(p,\lambda)$ of the input reflection function $S_{11}(p,\lambda) = h(p,\lambda)/g(p,\lambda)$. The mixed element structure is assumed to be separable into its lumped and distributed parts which can in turn completely be defined in terms of the corresponding h polynomials $h(p,0)$ and $h(0,\lambda)$, provided that the polynomials $f(p,0)$ and $f(0,\lambda)$ are defined.
- Starting from the arbitrary, unconstraint coefficients of $h(p,0)$ and $h(0,\lambda)$, we generate the remaining unknown coefficients of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$ by utilizing the connectivity information supplied for the cascade structure.
- Then, the coefficients of the polynomials $h(p,0)$ and $h(0,\lambda)$ are chosen as the independent unknowns of the problem and determined to optimize the gain of the system by means of a nonlinear search routine.

In the above procedure, there is no restriction on the unknown real coefficients of the polynomials $h(p,0)$ and $h(0,\lambda)$. Therefore any unconstraint optimization routine can be employed. The numeric in the optimization of gain function is well behaved and the convergence becomes much faster than the direct optimization of the element values since the transducer gain of the structure written in terms of coefficients of $h(p,\lambda)$ is quasi-quadratic. Once the final forms of $g(p,\lambda)$ and $h(p,\lambda)$ are generated, the mixed element realization can be obtained by employing the algebraic decomposition technique on the polynomial sets describing the lumped and distributed prototypes.

It is straightforward to extend the SRFT to design multi-stage microwave amplifiers by generating the TPG in a sequential manner.

IV. EXAMPLES

As an example, a double stage FET amplifier is designed for 50 Ohm terminations. The proposed mixed lumped and distributed circuit models are utilized as front-end, back-end and interstage equalizers. The measured scattering data available for the active device pair of HP 1-μm gate packaged microwave transistors are directly processed to obtain an octave band amplifier over the frequency band of 4-8 GHz. As a result of design process, the obtained matched amplifier system has an average gain level of 13.5 dB over the design frequency band. The design steps and the considerations on the final circuit realization will be discussed due to space limitations in the conference.

V. CONCLUSION

A computer aided design technique for broadband microwave amplifiers employing distributed equalizers with lumped discontinuity models is presented. The method is based on the two variable description of lossless matching equalizers on a scattering basis. For the generation of scattering functions in two complex frequency variables a semianalytic approach is deviced and extended to the real frequency design of lossless equalizers of broadband amplifiers. The possibility of incorporating the studied regular mixed lumped-distributed topologies to model the discontinuities and interconnects of a disrtibuted design is discussed and the use of the method is illustrated by design examples.

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